To Mask or Not to Mask

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Abstract

Some governments implement mandatory mask policies partly based on the scientific studies that show mask-wearing helps "flatten the curve." Taking mask-wearing as exogenous behaviors makes these studies unable to tell when and why people would comply. I endogenize individuals' mask-wearing decision in a model in which selfish individuals know that masks protect others more. Their equilibrium decisions exhibit inter-dependence. A parameter that proxies the population density determines whether mask-wearing are substitutes or complements among individuals. Without relying on behavioral assumptions and ad hoc differences, the model offers a rational explanation of the polar opposite cases among equally-crowded cities: some in which almost everyone wears masks, but few do so in others. Comparing social and private incentives, the model identifies the scenarios wherein mandatory mask policies benefit the society and wherein people comply with such policies. It highlights how economics differs from science in calculating the effectiveness of mask-wearing in containing the virus.

Keywords: Mask; population density; public good; COVID-19; infection.

JEL Classifications: H30, H41, I18, R11

1. Introduction

Everyone knows we need vaccines with manageable side effects badly. Most agree social-distancing can slow down the spread of the virus. Few doubt the importance of fast and cheap test kits. However, the world remains divided with regard to mask wearing, with the debate continuing between mask wearers and those who swear they will never wear one. Existing public policies are equally divided, even within a country.[1]

[1] Feng et al. (2020) conduct a recent survey of the recommendations and policies across different places concerning the use of face masks. The U.S. Center for Disease Control and Prevention and the Singaporean government did not recommend mask wearing in public until early April. Abaluck et al. (2020) carefully sort out the countries in their sample that changed their mask policies, including Switzerland, Austria, Czech, Australia, Romania, Thailand, Bulgaria, and Singapore.

The controversy regarding mask wearing makes sense in economics. Apart from the fact that masks are costly and uncomfortable to wear, wearers offer more protection to others than themselves.[2] Some policymakers encourage people to wear masks while at the same time reminding them that doing so protects others but probably not themselves.[3] Such externalities make mask wearing a classic public good: non-rivalrous in a sense that one nearby getting protected does not diminish the mask's protective effects on others; non-excludable in a sense that it is difficult, if not impossible, to charge the nearby for your wearing a mask.

Why would people in some places choose to wear masks? Do they simply misunderstand how masks work? Did they get the math wrong and miscalculate the risks? Do they care and thus act more responsive to the #StopTheSpread hashtag than others? Are they overly cautious? Is their action simply a political act against the advice of the government or the WHO? Do non-mask wearers feel social pressure from mask wearers? Does signaling to others that you care matter during such difficult times?

Although these explanations are not necessarily incorrect, they cannot help address the important policy question: As masks help "flatten the curve" only when most people wear one, do individuals acting in their self-interest have an incentive to do so? Otherwise, under what conditions does a mandatory mask wearing policy make sense? Would people comply? When should we expect few would comply, leading to huge enforcement costs?

To address these policy issues, I build a model that rules out all these behavioral assumptions to find out the scenarios in which it is in the self-interest of rational and well-informed individuals to wear a mask that costs them something and benefits others more. The model has the following few key ingredients.

1. **Externalities**. Wearing a mask protects others, but it is impossible for mask wearers to charge them.

[2] Hamsters have helped prove this point. Chan et al. (2020) place infected and healthy hamsters in separate cages. Air was blown from the former to the latter. The infection rate after a week depended on how surgical masks were placed: 66.7% if not placed at all, 33.3% if placed on the cage of the healthy hamsters, and 16.7% if placed on the cage of the healthy hamsters. These infection rates are discussed in the simulations in Section 5.

[3] Canada's top health officer Dr. Theresa Tam reminded Canadians in early April, "Wearing a non-medical mask is an additional measure that you can take to protect others around you," She warned, however, that a non-medical mask does not necessarily protect the person wearing it. Tasker, John Paul (2020 April 6) "Canada's top doctor says non-medical masks can help stop the spread of COVID-19" *CBC News* Retrieved from https://www.cbc.ca/news/politics/non-medical-masks-covid-19-spread-1.5523321

2. **Weak protection**. Masks are meant for those who are sick. It offers some but limited protection to healthy people.[4] If a pair of infected and non-infected persons bump into each other, the virus spreads much slower if the infected person wears the mask instead of the healthy one.

3. **Zero protection**. To those already infected (i.e., asymptomatic), wearing a mask only prevents them from infecting others and does not benefit themselves.

4. **Asymptomatic and presymptomatic infections**. A key difficulty in dealing with the new virus is its undetected spread: an infected person without symptoms can still infect others (He et al., 2020). A person has to decide whether or not to wear a mask even without knowing if she is already infected.

5. **Self-interest**. People do not derive utility from protecting others or others' health. They only care about their own health.

6. **No misinformation**. Everyone knows how masks work.

These ingredients are put into play in a strategic game in which each player decides whether or not to wear a mask. A key driver in the model is the number of individuals that one person randomly "bumps" into; I regard such scenario as inevitable in our daily life. The word "bump" here does not strictly refer to seeing and interacting with someone directly. It can mean taking an elevator, riding a bus or train, or entering an enclosed area (such as a public toilet) that others have used previously, thereby resulting in an infection. The science lies in the fact that virus transmission can be airborne, that is, a virus stays in the air even after an infected person leaves the area. Scientific studies find that coughing, sneezing, and simply breathing and talking can spread the virus; however, their findings regarding flatulence are not conclusive. These actions create droplets that can hang in the air for a certain period. One way to understand why lockdown reduces the spread of viruses is that it abruptly cuts down the number of individuals inevitably bumping into one another. While one may interpret this driver as population density, the two notions are not exactly the same. One caveat of the model is that this driver is *not* endogenous.

[4] The weak protection provided by masks can be understood as a reduction in the chance of getting infected by being around an infected person. Suppose that such chance is 90% if one is not wearing a mask and 70% if one wears one; the reduction in this case is 20% only.

I show the cost and benefit trade-off of wearing a mask. The benefit (denoted as MB for marginal benefit), simply put, is the increase in the probability of staying healthy.[5] I find that the magnitude depends not only *scientifically* on the filtration efficiencies of masks to and from the wearers but also *economically* on how many others are infected and whether they wear masks too, as well as how many people are inevitably "bumping" into others in non-trivial ways.

While fully acknowledging the presence of monetary and non-monetary costs, I lump all the costs of wearing a mask into one parameter denoted by c.[6]

If each individual's MB outweighs c, then in equilibrium, everyone wears a mask, and vice versa. A mixed-strategy equilibrium may occur, in which case everyone wears a mask with a certain probability.[7]

I show that despite weak protection and the public good nature of mask wearing, selfish individuals may rationally choose to wear masks even though doing so benefits others more. I find that free-riding on others to wear masks only happens when one does not have to "bump" into many people; if one has to, mask wearing exhibits strategic complementarity, that is, more people wearing masks incentivizes one to wear one.

If one takes the view that people in a crowded place cannot avoid "bumping" into many people, then the model shows that mask wearing by everyone and mask wearing by no one can be equilibria in a croweded place. The model thus offers an economic explanation for the difference between Hong Kong (where everyone wears masks (Cowling et al., 2020)) and other equally crowded places such as Manhattan (where only a few wears a mask) without assuming ad hoc differences.

The recent changes in public policies (both compulsory ones and those resorting to people's

[5] This increase in probability multiplied by the payoff difference between staying healthy and getting infected (which I normalize to 1) is the benefit of wearing a mask in the game.

[6] In addition to feelings of discomfort (especially for those with beards), searching and queuing for masks are costs. Other non-monetary costs include the steep learning curve of wearing a mask properly. Mistakes include wearing masks upside down, inside out, with the nose exposed, and with the tin left unbent and not fitting the shape of the face; touching the mask; lowering the mask to cough/sneeze (unfortunately, I witnessed many people doing so, probably because they do not want to make their masks dirty); talking on the phone with the mask lowered; forgetting to pull the mask all the way down to fully cover the chin; and inappropriate sizing, resulting in large gaps. The worst mistake is probably reusing a mask too many times (which I must admit I have done in the past when I did not have enough masks left at home). Other issues include using low-quality masks with compromised filters, learning the differences between the different types of filter (KF94, KF99, BFE, PFE, VFE, different levels of ASTM, EN14683, etc.), and learning how to detect the validity of masks' quality certification.

[7] The stability of a mixed-strategy equilibrium, however, depends on the number of people that one inevitably "bumps" into.

emotions, such as using the #StopTheSpread hashtag in social media), which advise people to wear masks, can be understood as ways to "refine the equilibrium" from the no-one-wears-masks scenario to the everyone-wears-masks scenario.

2. Model

Players: Consider N persons in an economy, where N is a considerably large positive integer. At the beginning of the game, each person can be either infected already (with probability α) or healthy (with probability $(1-\alpha)$), where $\alpha \in [0, 1]$. I should note that no person in this case knows whether he or she is already infected.

Actions: Each person decides whether or not to wear a mask. Wearing a mask costs them c, where $c > 0$. To allow randomization, let us denote the probability of wearing a mask as $q \in [0, 1]$. Therefore, if $q = 1$, then the person always wears a mask; if $q = 0$, then the person never wears a mask. If q is between 0 and 1, then the person wears a mask only with probability q.

Payoffs: At the end of the game, if one does not wear a mask and remains healthy, then the payoff is 1; otherwise, the payoff is 0. If one wears a mask and remains healthy, then the payoff is $1 - c$; otherwise, the payoff is $-c$.

Beliefs: Each person believes that either herself or a random person is already infected with a probability α. Thus, their beliefs are consistent with the underlying environment.

Interactions: Assume that each person inevitably "bumps" into $M \in \{0, 1, 2, ...\}$ persons randomly during the game. As described in the introduction, "bump" here does not mean a direct interaction; they can indirectly interact by sharing an enclosed area within a window of time without directly seeing each other.

Transmission: If a healthy person "bumps" into an infected person, the probability of such healthy person staying healthy (i.e., not getting infected) depends on whether or not she and the infected person wear masks. The notations of these probabilities are shown in Table 1.

Table 1: Transmission: probabilities of a healthy person staying healthy after "bumping" into an infected person

		Healthy	
		Mask	None
Infected	Mask	i	j
	None	k	l

As i, j, k, and l are all probabilities, they are bounded below by 0 and above by 1. Let us now make further assumptions on these probabilities.

Assumption 1 $0 < l < k < j < i < 1$.

Assumption 1 means that infection is least (most) likely to happen if both persons (no one) in the scenario wear(s) masks. The inequalities $i > j$ and $k > l$ mean that a mask always provides some form of protection. The inequality $j > k$ means that a mask prevents infection more effectively when the infected person wears it than when the healthy person wears it.

Assumption 2 $i - j < k - l$.

Assumption 2 means that a mask protects a person more if the infected one does not wear one than if the infected one wears one.

How long is the game? Although one can get infected by "bumping" into an infected person, being infected during the game does not allow one to further infect others. The corresponding biological concept is the amount of time it takes for an infected person to start the asymptomatic shedding of enough viruses to infect others. One can understand the game as a snapshot in time long enough for asymptomatic viral shedding but shorter than needed for anyone to show symptoms.[8]

The solution concept is Nash equilibrium. A Nash equilibrium is an action profile $(q_1, q_2, ..., q_N)$, where the subscript indexes a person such that no one has an incentive to deviate given the actions of others. Although multiple equilibria can occur in the game, with some of them being asymmetric, I focus only on symmetric equilibria to substantially simplify the analysis and allow the equations to be intuitive.[9]

[8]Generally, a quarantine period of 14 days is deemed long enough for most people to show symptoms. Therefore, the game can last less than 2 weeks.

[9]Cabral (1988) shows that in symmetric games with many players, one can understand an asymmetric pure-strategy

2.1. *Individual benefit-and-cost analysis*

Normalizing to 1 the payoff difference between staying healthy and getting infected allows one to interpret both the marginal benefit (MB) and marginal cost of wearing a mask as a percentage of the difference between staying healthy and getting infected. Therefore, the model can be used for comparison across places that potentially differ in terms of per-capita income and other dimensions.[10]

The marginal cost of wearing a mask is c, i.e., $MC = c$.

The MB of wearing a mask is the resulting increase in the probability of staying healthy (times 1, the payoff difference). Focusing on symmetric equilibria means that for a player, all other players wear masks with the same probability. Denote such a probability for everyone else as q. This approach greatly simplifies the analysis and allows the MB to be intuitive.

With a mask, the probability of staying healthy when a healthy individual randomly "bumps" into M people is

$$\Pr(\textit{Healthy}|\textit{Mask}) = [(1-\alpha) + \alpha(qi + (1-q)k)]^M. \tag{2.1}$$

The first term inside the square bracket is the chance that a random person is healthy; the second term refers to the chance that a random person is already infected but that the infection does not happen. With probability q, the infected person is wearing a mask; in such a case, the chance of staying healthy is i. With probability $(1-q)$, the infected person has no mask; in such a case, the chance of staying healthy is k. The greater the number of people that one person randomly "bumps" into, the lower the chance of this one person to stay healthy; this relation is captured by "to the power M" due to statistical independence.

Similarly, with no mask, the probability of staying healthy when a healthy individual

equilibrium as an approximate outcome of the play of a specific symmetric mixed-strategy equilibrium. For instance, if the population size is large enough, then an individual expecting that only 1 in 3 people wears a mask (clearly an asymmetric equilibrium) can also view everyone as having a one-third chance of wearing a mask. Therefore, the focus of symmetric equilibria instead of all equilibria does not appear to incur much loss while the clarity improvement is huge.

[10]For instance, one can say to a Torontonian that the cost of a mask is roughly 4% of the payoff difference between staying healthy and getting infected; such value can be roughly in the same neighborhood as that for a typical New Yorker.

randomly "bumps" into M people is

$$\Pr(Healthy|None) = [(1-\alpha) + \alpha(qj + (1-q)l)]^M. \tag{2.2}$$

With probability $(1-\alpha)$, a player begins the game as a healthy individual. The MB of wearing a mask is thus $(1-\alpha)\big[\Pr(Healthy|Mask) - \Pr(Healthy|None)\big]$; Lemma 1 sums up the calculation.

Lemma 1 *An individual's marginal benefit of wearing a mask is*

$$\begin{aligned} MB(q, \alpha, M, i, j, k, l) =& (1-\alpha)\{[(1-\alpha) + \alpha(qi + (1-q)k)]^M \\ & - [(1-\alpha) + \alpha(qj + (1-q)l)]^M\} \end{aligned} \tag{2.3}$$

Equation (2.3) indicates that under assumption 1, $MB > 0$ for any finite $M > 0$. Regardless of how weak the protection offered by a mask to the wearer is, as long as c is low enough, it is in the self-interest of selfish individuals to wear masks. Equation (2.3) embeds two interesting mathematical properties of MB; I sum them up in the following two lemmas. The proofs are presented in the Appendix.

Lemma 2 *With sufficiently few infected people (i.e., small α), MB first increases in M and then decreases in M.*

Lemma 3 *There exists a range of $(\underline{M}, \overline{M})$ such that*
(a) for any M below this range, MB decreases with q;
(b) for any M above this range, MB increases with q;
(c) for any M within this range, MB first increases then decreases with q.

The intuition of Lemma 2 is as follows: A mask matters very little to a person if the risk of getting infected is extremely low or extremely high. One can see it mathematically as $MB = 0$ when $M = 0$ (zero risk of getting infected by others) and $M = \infty$ (the risk of getting infected is 1). As M increases, the infection risk rises and becomes inevitable; thus, MB is non-monotonic in relation to M, that is, it first increases and then decreases.

The intuition of part (a) of Lemma 3 is free-riding: Under small M, one has *less* incentive to wear a mask when more people wear masks. Such a free-riding incentive comes from the fact that wearing masks is a strategic substitute among individuals that creates problems in the current pandemic: for masks to help "flatten the curve," everyone should wear them. However, if everyone else is wearing a mask, it makes perfect sense for one not to. The good news is that such free-riding only happens up to a certain point.

The intuition of part (b) of Lemma 3 is strategic complementarity: Under large M, one has *more* incentive to wear a mask when more people wear masks. I am not aware of any work mentioning this less obvious incentive in this pandemic: a large M means a high infection risk. When few wear masks, one wearing a mask helps that person very little; the risk of getting infected remains high. Increasing the number of other people wearing masks moderates the infection risk, thus incentivizing a person to also wear a mask to stay healthy.[11]

2.2. *Equilibrium characterization*

I formally characterize the equilibrium here. The intuition is simple: in equilibrium, a player wears a mask when $MB \geq c$; she does not wear one when $MB < c$. This player randomizes only when $MB = c$.

2.2.1. *Pure-strategy Nash equilibrium*

Under what conditions would everyone and no one wearing a mask be equilibrium?[12]

Proposition 1 *Action profile*

(a) $(1, 1, ..., 1)$ *is a pure-strategy Nash equilibrium if* $c \leq MB(1, \alpha, M, i, j, k, l)$*;*

(b) $(0, 0, ..., 0)$ *is a pure-strategy Nash equilibrium if* $c \geq MB(0, \alpha, M, i, j, k, l)$*.*

The intuition is that whenever the cost of wearing a mask is sufficiently low, it is in the self-interest of everyone to wear a mask, and vice versa.

In this pandemic, everyone wears a mask in some but not all crowded areas. Many people in some crowded areas do not wear masks unless it is mandatory. Can the model tell us why? In other words, can both action profiles$(0, 0, ..., 0)$ and $(1, 1, ..., 1)$ be equilibria at the same time? To address this question, I define three thresholds of M.

[11] One can think of part (c) of Lemma 3 as being a necessary mathematical transition from the freeriding range to the herding range.

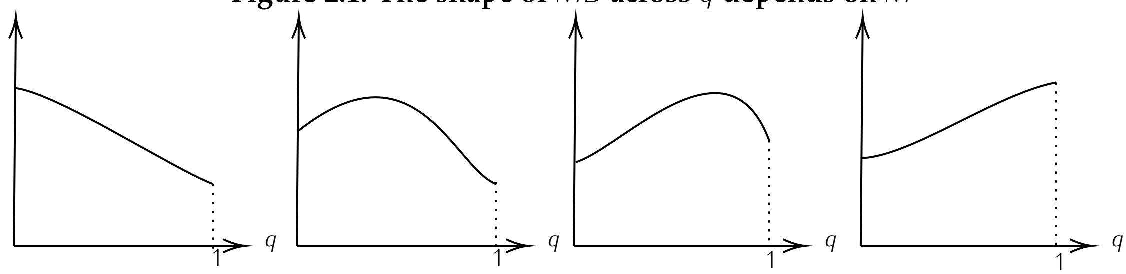
[12] Proving both Propositions 1 and 2 is simple. Given everyone else's mask-wearing probabilities, no one can increase her payoff by changing his or her mask-wearing probability.

Define $\underline{M} \equiv \arg\sup_M \{M \in \mathbb{Z}^+ | \frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q} < 0\} \; \forall q \in [0,1]$.

Define $\overline{M} \equiv \arg\inf_M \{M \in \mathbb{Z}^+ | \frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q} > 0\} \; \forall q \in [0,1]$.

Define $\widehat{M} \equiv \arg\sup_M \{M \in \mathbb{Z}^+ | MB(0,\alpha,\widehat{M},i,j,k,l) > MB(1,\alpha,\widehat{M},i,j,k,l)\}$.

Figure 2.1: The shape of MB across q depends on M



(a) $M \leq \underline{M}$ (b) $\underline{M} < M \leq \widehat{M}$ (c) $\widehat{M} < M < \overline{M}$ (d) $\overline{M} \leq M$

Figure 2.1 visualizes Lemma 3 under these thresholds and shows that if $M > \widehat{M}$, then $MB(0,\alpha,M,i,j,k,l) \leq MB(1,\alpha,M,i,j,k,l)$. If the cost of wearing a mask c falls within $MB(0,\alpha,M,i,j,k,l)$ and $MB(1,\alpha,M,i,j,k,l)$, then the scenarios where everyone and no one wears a mask are equilibria. I summarize this result in the following proposition.

Proposition 2 *For $M > \widehat{M}$, if $MB(0,\alpha,M,i,j,k,l) \leq c \leq MB(1,\alpha,M,i,j,k,l)$, then both action profiles $(0,0,...,0)$ and $(1,1,...,1)$ are equilibria.*

Proposition 2 happens in relatively crowded places where M is sufficiently large. The model thus offers a plausible explanation for the polar opposite cases among different crowded areas *without* the need to assume any exogenous differences across areas. Proposition 2 suggests that there is nothing inherently irrational for people in different crowded places to behave differently.

Proposition 2 also helps justify mandatory face covering policies, such as Maryland's executive order issued by Governor Larry Hogan on April 15, 2020.[13] If it is reasonable to expect that pair (M,c) happens to fall within such a parameter space, then the mandatory order helps "refine the equilibrium" away from the no-one-wears-masks scenario to the everyone-wears-masks scenario. If there is no reason to believe that pair (M,c) is within such a parameter space, then the mandatory order is incentive-incompatible, which likely results in high enforcement costs and low compliance rates.

[13]On April 15, 2020, Maryland Governor Larry Hogan issued a mask and physical distancing order. The order can be retrieved from https://governor.maryland.gov/wp-content/uploads/2020/04/Masks-and-Physical-Distancing-4.15.20.pdf

2.2.2. Mixed-strategy Nash equilibrium

The players may randomize in equilibrium when c is equal to MB for some q between 0 and 1, as stated in the following Proposition. The model can thus explain those areas where only a few people wear masks. Whereas economists may find those equilibria interesting, policymakers and scientists from other fields may skip the following details without losing much of the big picture.

Proposition 3 *If $c = MB(q^*, \alpha, M, i, j, k, l)$ for $q^* \in (0, 1)$, then action profile $(q^*, q^*, ..., q^*)$ is a mixed-strategy Nash equilibrium.*

The existence depends on the pair (M, c). As shown in Figure 2.1, one such mixed-strategy Nash equilibrium exists in the following three situations.

(a) For $M \leq \widehat{M}$, it is when $MB(1, \alpha, M, i, j, k, l) < c < MB(0, \alpha, M, i, j, k, l)$;

(b) For $M > \widehat{M}$, it is when $MB(0, \alpha, M, i, j, k, l) < c < MB(1, \alpha, M, i, j, k, l)$;

(c) For $\underline{M} < M < \overline{M}$, it is when $c = max_q MB(q, \alpha, M, i, j, k, l)$

Two such mixed-strategy Nash equilibria exist in the following two situations.

(d) For $\underline{M} < M \leq \widehat{M}$, it is when $MB(0, \alpha, M, i, j, k, l) < c < max_q MB(q, \alpha, M, i, j, k, l)$;

(e) For $\widehat{M} < M < \overline{M}$, it is when $MB(1, \alpha, M, i, j, k, l) < c < max_q MB(q, \alpha, M, i, j, k, l)$.

A side-note is that when MB is increasing in q, wearing masks becomes a strategic complement among people, thus rendering any mixed-strategy equilibria unstable if q^* happens to be at the upward-sloping part of the corresponding MB.[14]

3. Social versus private incentives

The results show that even though masks are more about protecting others, it can be in the self-interest of selfish individuals to wear masks even if they know that wearing masks protects others more.

Some policymakers are aware of the fact that mask wearers protect others.[15] This fact

[14] Those described in (b) and the one with smaller probabilities in (d) and (e) are thus unstable. Echenique and Edlin (2004) prove that strict strategic complementarities make the mixed-strategy Nash equilibrium unstable.

[15] Policymakers from both Canada and the U.S. seem to be aware. "Wearing a non-medical mask is an additional measure that you can take to protect others around you," Canada's Chief Public Health Officer Dr. Theresa Tam said on early April, reversing her advice against masks. She warned, however, that a non-medical mask does not necessarily protect the person wearing it. Tasker, John Paul (2020 April 6) "Canada's top doctor says non-

may explain why when discussing his executive order, Governor Larry Hogan elevated "wearing masks" to a new moral high ground by arguing that not doing so infringes others' rights:

> "Some people have said that covering their face infringes on their rights. This isn't just about your rights or protecting yourself. It's about protecting your neighbors, and the best science that we have shows that people might not know that they're carriers of the virus and through no fault of their own, they could infect other people. Spreading this disease infringes on your neighbor's rights."[16]

To justify mandatory mask wearing policies, one needs to know when the corresponding private incentives are too weak.

3.1. When would universal mask wearing make the society better off than no mask wearing?

The aggregate cost of everyone wearing a mask is Nc. The aggregate payoff increase is N times the difference between the probability of staying healthy when everyone wears a mask and the probability of staying healthy when no one wears a mask (again, times 1, the payoff difference).

With a mask, the probability of staying healthy when a healthy individual randomly "bumps" into M people under universal mask wearing is

$$\Pr(\textit{Healthy}|\textit{Universal}) = [(1-\alpha) + \alpha i]^M. \tag{3.1}$$

Similarly, when no one wears a mask, the probability of staying healthy when a healthy individual randomly "bumps" into M people is

$$\Pr(\textit{Healthy}|\textit{None}) = [(1-\alpha) + \alpha l]^M. \tag{3.2}$$

An individual's social MB of wearing a mask is $(1-\alpha)[\Pr(\textit{Healthy}|\textit{Universal}) - \Pr(\textit{Healthy}|\textit{None})]$

medical masks can help stop the spread of COVID-19" *CBC News* Retrieved from https://www.cbc.ca/news/politics/non-medical-masks-covid-19-spread-1.5523321

[16] Miller, Stetson (2020 Apr 18) "Coronavirus Latest: Executive Order Requiring Face Coverings In All Maryland Businesses, Public Transit Goes Into Effect" *CBS Baltimore* Retrieved from https://baltimore.cbslocal.com/2020/04/18/coronavirus-latest-face-coverings-executive-order-maryland

or:

$$MB^*(\alpha, M, i, l) = (1-\alpha)\{[(1-\alpha)+\alpha i]^M - [(1-\alpha)+\alpha l)]^M\}. \tag{3.3}$$

By assumption 1, $MB^*(\alpha, M, i, l) > MB(q, \alpha, M, i, j, k, l)$ for all $q \in [0,1]$. By Proposition 1, the social incentive for universal mask wearing is always higher than the corresponding private incentive. If c somehow falls between $MB^*(\alpha, M, i, l)$ and $MB(1, \alpha, M, i, j, k, l)$, then we should not expect voluntary universal mask wearing in equilibrium even though it yields higher social surplus. I summarize the concept in the following proposition.

Proposition 4 *Universal mask wearing yields higher social surplus than no mask wearing when* $c < MB^*(\alpha, M, i, l)$*, where* $MB^*(\alpha, M, i, l) > MB(q, \alpha, M, i, j, k, l)$ *for all* $q \in [0,1]$.

Examining the infection rates of 42 countries differ in terms of the norms of mask wearing, Abaluck et al. (2020) recommend universal mask wearing. The authors expressed concern about the divergence of the private and social benefits of mask wearing and advocated the emphasis on the social benefits of mask wearing so as to motivate more people to wear masks. Proposition 4 echoes their view.

Such a divergence of social and private incentives of universal mask wearing may help one make a case for compulsory mask wearing. The model, however, calls for a careful estimate of the enforcement costs as it could be high due to incentive incompatibility.

Note that mathematically speaking, nothing precludes the social optimum to be such that the probability of wearing a mask falls somewhere between 0 and 1.[17] Nevertheless, aligning it with the policy perspective is difficult. I therefore skip this complication.

[17] A social planner's problem is

$$max_q N\Big[(1-\alpha)\big[(1-\alpha)+\alpha(q^2 i + q(1-q)(j+k) + (1-q)^2 l)\big]^M - cq)\Big].$$

Within a range of parameters, the socially optimal q can fall within 0 and 1. However, even if the social optimum is, say, 65%, a policymaker is not likely to mandate that only 65% of people wear masks. Enforcement means all or nothing.

4. Contagiousness: Economics versus science

The model suggests that the contagiousness of the virus is not totally objectively defined by science; it is an equilibrium concept determined by the equilibrium probability for people to wear masks in a somewhat less trivial way.[18] Therefore, suggesting the importance of wearing masks matters simply by looking at the filtration efficiencies of masks is incomplete without incorporating people's endogenous mask-wearing decisions.[19]

One equilibrium outcome of the model is the expected number of healthy people. At the beginning of the game with α already infected people, M, c, and the transmission probabilities determine the equilibrium $q^* = f(i,j,k,l,M,c)$. Among those $(1-\alpha)$ healthy individuals, each has the following probability of staying healthy:

$$\begin{aligned}\Pr(Healthy) =& q^*\Pr(Healthy|Mask) - (1-q^*)\Pr(Healthy|None)\\ =& q^*((1-\alpha)+\alpha(q^*i+(1-q^*)k))^M\\ &+(1-q^*)((1-\alpha)+\alpha(q^*j+(1-q^*)l))^M.\end{aligned} \tag{4.1}$$

Thus, the number of healthy individuals getting infected is $(1-\alpha)(1-\Pr(Healthy))$. Dividing this number by the number of already infected individuals yields the *economic* reproductive number in epidemiology:

$$\begin{aligned}R_0(q^*) =& \frac{1-\alpha}{\alpha}\Big[1-\big[q^*((1-\alpha)+\alpha(q^*i+(1-q^*)k))^M\\ &+(1-q^*)((1-\alpha)+\alpha(q^*j+(1-q^*)l))^M\big]\Big].\end{aligned} \tag{4.2}$$

The scientific benchmark, however, does not take mask wearing as an endogenous choice of an individual. Therefore, no equilibrium concept exists in the computation. Recall from Equations (3.1) and (3.2) the probabilities of each healthy individual of staying healthy when everyone wears

[18] In reality, a huge set of scientific studies proves the existence of a variety of different factors, particularly environmental factors, that determine the contagiousness of a virus.

[19] Tian et al. (2020); Kai et al. (2020) show computational models that quantify the impact of mask wearing on the contagiousness of the virus. However, people in their models do not choose whether or not to wear masks endogenously. Comparing Hong Kong to other crowded cities, Cheng et al. (2020) reports a negative relation between mask-wearing and infection rates.

a mask and when no one wears a mask. The *scientific* reproductive number in epidemiology is

$$R_0(1) = \frac{1-\alpha}{\alpha}\left[1 - \left[(1-\alpha) + \alpha i\right]^M\right], \tag{4.3}$$

if somehow everyone wears a mask and

$$R_0(0) = \frac{1-\alpha}{\alpha}\left[1 - \left[(1-\alpha) + \alpha l\right]^M\right], \tag{4.4}$$

if no one wears a mask. Clearly, $R_0(q^*) \in [R_0(0), R_0(1)]$.

The reduction of R_0 from $R_0(0)$ to $R_0(1)$ can be regarded as the scientific effectiveness of mask wearing in containing the spread. Economic modeling, however, shows that reaching this reduction by mask wearing is not guaranteed unless $q^* = 1$ is the equilibrium.

5. Simulations

I use the hamsters' infection rates mentioned in footnote 2 for the simulation. As no infection rate is available for both hamsters "wearing" masks, I make one up (6.7%, thus 93.3% is the probability of staying healthy). I also make up a worse set of infection rates to proxy homemade cloth face coverings, such as that recommended by the U.S. Surgeon General, and another better set. Table 2 shows the numbers.

I simulate the individual private MB of the different probabilities of everyone else wearing a mask in Figure 5.1 under four infection levels. I also simulate the social MB^*.

Table 2: Transmission: Probabilities of staying healthy for simulations

		Healthy					
		Better: Mask	Better: None	Hamster: Mask	Hamster: None	Homemade: Mask	Homemade: None
Infected	Mask	99%	90%	93.3%	83.3%	66.7%	56.7%
	None	70%	33.3%	66.7%	33.3%	46.7%	33.3%

The simulations yield the following results:

1. Consistent with Lemma 2, MB increases and then decreases, and it tends to 0 when M becomes large. One cannot guarantee that increasing M will encourage more people to wear

masks because MB does not increase monotonically with M.

2. Consistent with Lemma 3, MB decreases in q under small M and increases in q under large M.

3. MB is positive when $M > 0$, even for low-quality homemade masks (If c is low enough, at any positive M, the scenario with everyone wearing a mask is equilibrium.)

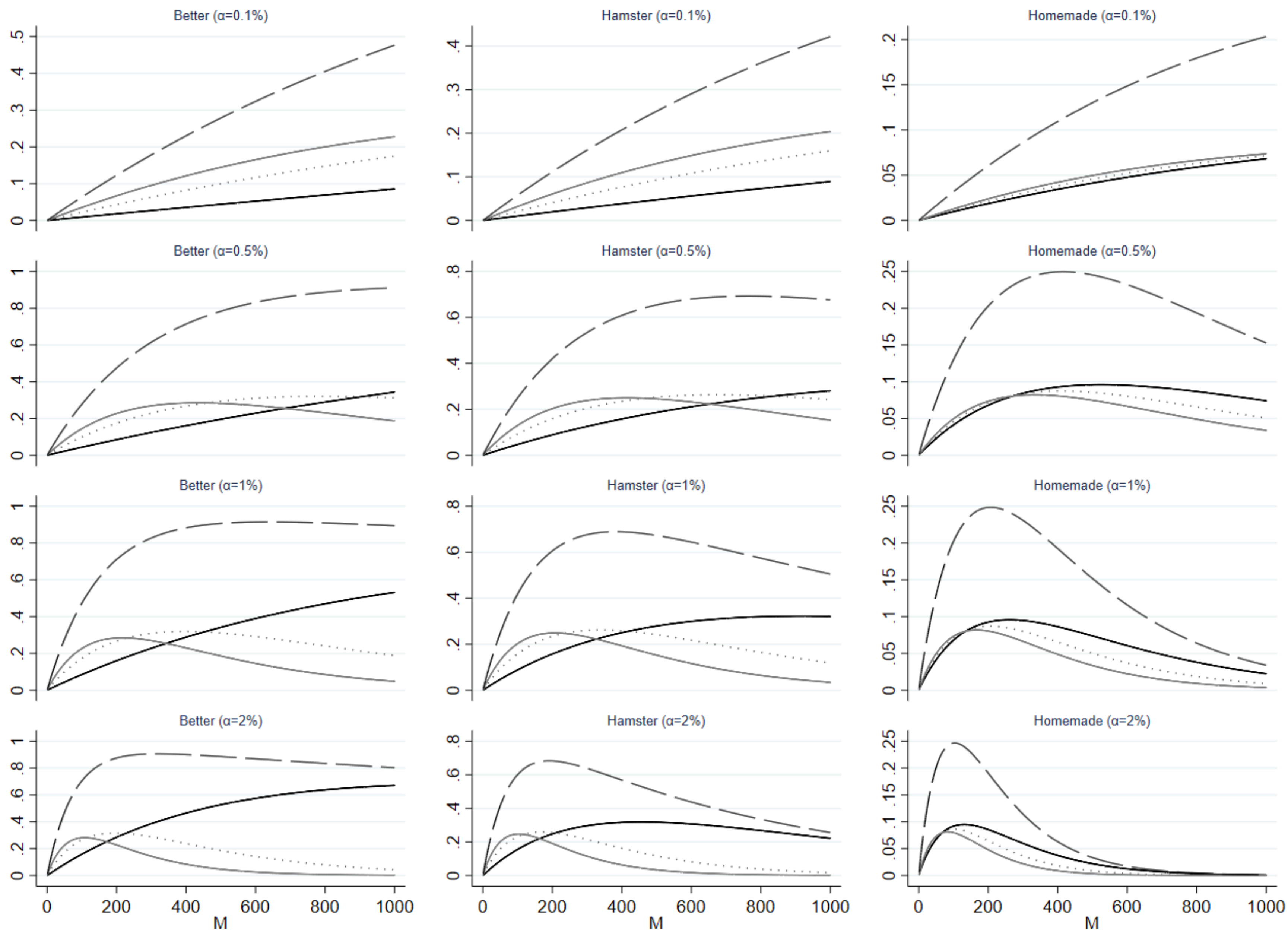
4. α affects MB non-monotonically. Therefore, one cannot claim that if the infection risk increases because more people are already infected, then more people will wear masks. It is only sometimes the case. At a certain infection level, the risk is too high for a mask to make a meaningful difference.

5. The range $(\underline{M}, \overline{M})$ drops when α increases.

6. Consistent with Proposition 4, $MB^* > MB(1, \alpha, M, i, j, k, l)$.

7. The divergence between social and private incentives can be substantial.

Figure 5.1: Simulations with 4 α and 3 mask quality levels: (MB of $q = 0$(gray), $q = 1/2$(dotted), $q = 1$(black), MB^* (dashed)



6. Policy implications

6.1. *Mandatory mask wearing policy*

Should masks be made mandatory in public areas? It depends on c. It is not necessary if the scenario in which everyone wears a mask is in equilibrium. If no one does so, then the model suggests to estimate c and M. If the pair falls in a region where the no-one-wears-masks scenario and the everyone-wears-masks scenario are equilibria, then a mandatory policy helps tilt the equilibrium from the former to the latter.

However, when c is higher than the private incentive for an individual to wear a mask when everyone else wears one but lower than the social incentive, even if it is socially more desirable for everyone to wear masks, the compliance of any individual is incentive-incompatible. The resulting enforcement costs are likely high.

Any mandatory mask wearing policy must ensure that the cost of wearing a mask is low enough.

6.2. *Quality policy*

The simulations show that high-quality masks do make a considerable difference. However, this result does not mean that low-quality masks do not make any difference. This outcome echoes scientific findings. Leung et al. (2020) show that even a low-quality, not particularly well-fitted mask is effective in trapping droplets from the wearer.

If masks are categorized as medical supplies and thus require approval from authorities before being sold, then mask supplies will be limited, and the cost of wearing a mask increases. Some have pointed out that the Food and Drug Administration's (FDA) categorization of masks as medical devices has slowed down the supplies of masks.[20]

One can understand U.S. Centers for Disease Control and Prevention's reversal from advising against wearing masks to recommending cloth face coverings on April 4, 2020 as a clever attempt to make the cost of wearing a mask as low as possible while getting around FDA

[20]Matzko, Paul. (2020 Apr 1) "To help solve the surgical mask shortage, get the FDA out of the way." *New York Daily News* Retrieved from https://www.nydailynews.com/opinion/ny-oped-surgical-masks-fda-20200401-vlwe72h76bb53hibyf5ddu6mou-story.html

regulations.

The economics is that being strict over quality reduces the availability and raises the cost of wearing masks. Not being too hung up with quality but instead focusing on making masks cheap to wear and widely available appears likely for mask wearing to be in the people's self-interest.

6.3. Other mask-related policy

Germany and Taiwan temporarily banned the exportation of face masks at the beginning of the pandemic to make sure that the local demands for masks could be met. The Taiwan government rationed masks at the beginning and used technology apps to track mask inventories. Anti-price gouging regulations were entertained by a number of Hong Kong politicians, but none was adopted. Appealing to people to "help protect others" lowers the psychological costs of wearing a mask. A variety of these mask-related policies can raise or lower the cost of wearing masks.[21]

An interesting episode happened in Czech Republic: Petr Ludwig, a key opinion leader, made a video on March 14, 2020, to discuss the rationale of wearing masks; this video went viral.[22] The influential video may have been instrumental in normalizing mask wearing in Czech Republic.

7. Concluding remarks

In this work, I build a model in which selfish individuals do not derive utility from protecting others. They know that wearing masks protects others. The model focuses on their individual choice of wearing masks, which is not endogenous in other scientific simulations. Under certain parameters, the model generates multiple equilibria, thereby offering a rational explanation for the different mask-wearing adoption rates observed in different crowded places.

The model is capable of deriving the condition under which mask wearing is in the self-interest of individuals. It allows us to see the divergence between the social and private incentives of universal mask wearing. It also highlights the role played by two important factors, namely,

[21] I am aware of Oxford's systematic collection and up-to-date policy measures to tackle COVID-19 and the resulting Stringency Index (Hale et al., 2020). However, their dataset does not explicitly collect the various mask policies used around the world. I have yet to locate a comprehensive database on mandatory mask-wearing and mask-related policies across countries.

[22] Abaluck et al. (2020) mention this interesting episode as well.

the cost of wearing a mask and the number of random individuals one inevitably "bumps" into in daily life. The latter proxies population density, although the two notions have their differences. The model allows us to predict whether or not people would comply with mandatory mask policies.

The model has a few shortcomings. First, M is not exogenous; to some extent, one usually has some leeway to control the number of random individuals that one person "bumps" into.

Second, changing M likely changes the payoff of an individual, but by how much is not clear. I am not aware of any estimate that precisely pins down the extent to which reducing M, such as drastic policies like stay-at-home orders, takes one's fun away. The model, therefore, cannot help derive the trade-off between mask-wearing policies and other drastic policies. Such trade-off is probably the most pressing concern given the prioritization of the re-opening of the economy.

Third, α already infected people are sometimes concentrated in certain places, such as elderly care centers in Denmark. Relative to younger individuals, the elderly are likely to die from the virus. The model has not incorporated any type of heterogeneity across people.

Fourth, the model is set up as a static model. It could be made dynamic such that one can use it to simulate trends. However, doing so requires assumptions about those infected during a given period, as well as the interaction of the cost of wearing masks across periods.

One future extension is to make use of the model to study the allocation of masks (and possibly other types of personal protective equipment [PPE]) across regions with varying availabilities of masks and PPE. This modification can be achieved by modeling more than one region facing similar mask-wearing problems. For example, although Manila is extremely crowded, the Philippines has other less crowded regions to consider. The infection rates across regions probably differ. Suppose that masks and other PPE are of limited supply. How should the government allocate them to achieve the most positive effects for the country? How would the socially optimal allocation depend on changing infection rates, crowdedness, and mask quality? Can this within-country allocation problem be applied across countries with different infection rates and other factors?

A. Proofs

Lemma 2 *With sufficiently few infected people (i.e., small α), MB first increases in M and then decreases in M.*

Proof Recall from (2.3) that MB is

$$\begin{aligned} MB(q, \alpha, M, i, j, k, l) =& (1-\alpha)\{[(1-\alpha) + \alpha(qi + (1-q)k)]^M \\ & - [(1-\alpha) + \alpha(qj + (1-q)l)]^M\}, \end{aligned}$$

Since $(1-\alpha)$ is a scalar, one can check how MB changes with M by checking how increasing M changes what's inside the curly bracket. Define $A \equiv [(1-\alpha) + \alpha(qi + (1-q)k)]$ and $B \equiv [(1-\alpha) + \alpha(qj + (1-q)l)]$.

Increasing M by 1 changes what's inside the curly bracket by $\{A^{M+1} - B^{M+1}\} - \{A^M - B^M\}$. This change is positive iff

$$(\frac{B}{A})^M > \frac{1-A}{1-B}. \tag{A.1}$$

Since by assumption 1, $0 < B < A < 1$, therefore $\frac{B}{A} < 1$, which means the left-hand-side of (A.1) must decrease in M monotonically. The right-hand-side of (A.1), $\frac{1-A}{1-B}$ is less than 1 and is a constant. Thus, if $(\frac{B}{A})^M > \frac{1-A}{1-B}$ when M is small, increasing M will eventually upset this inequality, which means increasing M further would decrease $\{A^M - B^M\}$.

The remaining part is checking whether a small α is sufficient for $\{A^M - B^M\}$ to first increase before decrease in M. It is the case when $\{A^2 - B^2\}$ is larger than $\{A - B\}$.

Since $\{A^2 - B^2\} = (A-B)[(A-B) + 2B]$, if $[(A-B) + 2B] > 1$, then $\{A^2 - B^2\}$ is larger than $\{A - B\}$. A sufficient condition is $B > \frac{1}{2}$. If $(1-\alpha) > \frac{1}{2}$, then $B > \frac{1}{2}$. Thus, when $\alpha < \frac{1}{2}$, $\{A^2 - B^2\} > \{A - B\}$. $\square$

Lemma 3 *There exists a range of $(\underline{M}, \overline{M})$ such that*

(a) for any M below this range, MB decreases with q;

(b) for any M above this range, MB increases with q;

(c) for any M within this range, MB first increases then decreases with q.

Proof The derivative of MB with respect to q is

$$\frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q} = (1-\alpha)M\alpha\{(i-k)[(1-\alpha)+\alpha(qi+(1-q)k)]^{M-1} - (j-l)[(1-\alpha)+\alpha(qj+(1-q)l)]^{M-1}\}.$$

For $M = 1$, $\frac{\partial MB(q,\alpha,1,i,j,k,l)}{\partial q} = (1-\alpha)\alpha[(i-k)-(j-l)]$, which is negative due to assumption 2. For $M > 1$, the sign of $\frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q}$ is the sign of what's inside the curly bracket. Rearranging terms, $\frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q} > 0$ iff

$$\frac{i-k}{j-l} > \Big[\frac{(1-\alpha)+\alpha(qj+(1-q)l)}{(1-\alpha)+\alpha(qi+(1-q)k)}\Big]^{M-1}. \tag{A.2}$$

By assumption 2, the value of the left-hand-side of (A.2) is smaller than 1 (i.e., $\frac{i-k}{j-l} < 1$). By assumption 1, the right-hand-side of (A.2) is a fraction smaller than 1 to the power $M-1$, which is monotonically decreasing in M. Therefore, when M passes a certain threshold $\overline{M}$, the right-hand-side of (A.2) must become smaller than the left-hand-side of (A.2). On the other hand, when M is small enough such that it is below a certain threshold $\underline{M}$, then the right-hand-side must be larger than the left-hand-side (which is certainly true when $M = 1$). Combined, I have proven parts (a) and (b).

What is happening between the two thresholds? It is the range of M such that $MB(q,\alpha,M,i,j,k,l)$ is transitioning from everywhere decreasing in q to everywhere increasing in q. This is the case when we have $(\frac{(1-\alpha)+\alpha j}{(1-\alpha)+\alpha i})^{M-1} > \frac{i-k}{j-l} > (\frac{(1-\alpha)+\alpha l}{(1-\alpha)+\alpha k})^{M-1}$. The first inequality means $\frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q}$ evaluated at $q = 1$ is negative. The second inequality means $\frac{\partial MB(q,\alpha,M,i,j,k,l)}{\partial q}$ evaluated at $q = 0$ is positive. This is what part (c) refers to. $\square$

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